ENGINEERING PHYSICS

UNIT-5

DIELECTRIC & MAGNETIC MATERIALS

Vibha Masti

Feedback/corrections: vibha@pesu.pes.edu

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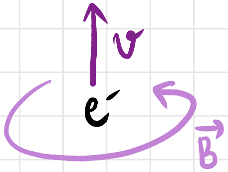
MAGNETISM

Origin of Magnetism

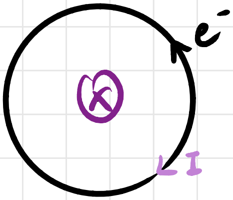
- Charges in motion responsible for magnetism



magnetism due to relativistic effects

Magnetic Dipole Moment

e^- in orbit



$$\mu = IA$$
$$= \frac{-e}{T}\pi r^2$$

$$\mu = \frac{-e}{2\cancel{\pi r}} v \cancel{\pi r^2} = \frac{-evr}{2}$$

$$\mu = \frac{-e}{2m} L$$

$$\vec{\mu} = \frac{-e}{2m}\vec{L}$$

$$\vec{\mu} = \gamma \vec{L}$$

$$\gamma = \frac{\mu}{L}$$

↑ gyromagnetic ratio

Quantum Picture

- $\vec{L}$ is quantised

$$L = \sqrt{l(l+1)}\,\hbar$$

$$L = n\hbar$$

$$\mu = \frac{-e}{2m}\,n\hbar$$

$n=1$

$$\mu_B = \frac{-e}{2m}\,\hbar$$

$$= 9.274 \times 10^{-24}\ \mathrm{J\,T^{-1}}$$

Bohr magneton

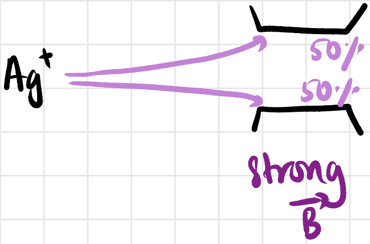
quanta of μ

- Spin from e^- also contributes to angular momentum — spin angular momentum

$$\mu_s = \frac{e}{2m}\sqrt{s(s+1)}\ \hbar$$

SWE of H-atom

Stern-Gerlach Experiment



something other than charge contributes

spin: either up or down

tiny magnet

$s = 1/2$ (spin quantum number – $m_s = \pm 1/2$)

- Two dipole moments interact – resultant

$$J = \vec{L} + \vec{S}$$

$$J = \sqrt{j(j+1)}\ \hbar$$ 2j + 1 orientations

total angular momentum

Landé g-factor

- Energy of system: interaction of $\vec{\mu}$ with $\vec{B}$

$$g_e = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}$$

$$\mu = \frac{e}{2m} g_e \hbar$$

Pure Orbital

g_e for $s=0$, $\vec{J} = \vec{L} \Rightarrow j = l$

$g_e = 1$ (no spin angular momentum)

$$\mu_{orb} = \frac{e\hbar}{2m}$$

Pure Spin

$l=0$, $\vec{J} = \vec{S} \Rightarrow j = s$

$g_e = 1+1 = 2$

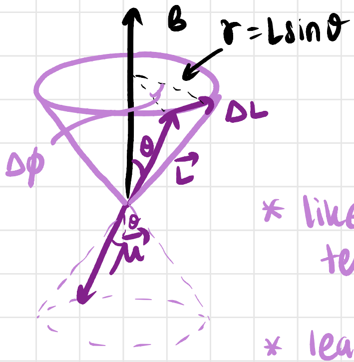
$$\mu_s = \left(\frac{e}{2m}\hbar\right) \times 2$$

- Therefore, spin dipole moment > orbital dipole moment
- significant contribution from spin
- All magnetic properties from spin (μ_s significant)

for any system

$$\mu = \frac{e}{2m}\hbar\, ge$$

Behaviour of Dipole in External Magnetic Field



$\vec{L}$ changes with time

* like cycle wheel with tension ↑

* learn about μ through L

- Torque on $\vec{L}$
- $\vec{L}$ starts moving — Gyroscopic precession

$$\tau = \frac{dL}{dt}$$

$$\vec{\mu} \times \vec{B} = \vec{\tau}$$

$$\mu B \sin\theta = \frac{dL}{dt}$$

$$\Delta L = L\sin\theta \, \Delta\phi$$

$$\frac{\Delta L}{\Delta t} = L\sin\theta \frac{\Delta\phi}{\Delta t}$$

$$\lim_{\Delta t \to 0} \frac{\Delta L}{\Delta t} = \frac{dL}{dt} = L\sin\theta \frac{d\phi}{dt}$$

frequency ω_L

$$\mu B\sin\theta = \frac{dL}{dt} = L\sin\theta \, \omega_L$$

Larmor frequency

$$\mu B = L\omega_L$$

gyromagnetic ratio

$$\omega_L = \frac{\mu}{L} B = \gamma B$$

$$\omega_L = \frac{e}{2m} B$$

- Dipole moment in $\vec{B}$ does not align, but precesses about it at a frequency

for $B = 1T$

$$\omega_L = 8.79 \times 10^{10} \text{ rad } T^{-1} s^{-1}$$

$$f_L = \frac{e}{4\pi m} B$$

14 GHz T^{-1}

for $B = 1T$

$$f_L = 1.4 \times 10^{10} \text{ Hz } T^{-1}$$

- Analysis can be repeated on nucleus — nuclear dipole moment (proton)
- Relative strength of dipole moment smaller

$$\mu_{nu} = \frac{e}{2m_p} B$$

- Does not contribute much to dipole moment
- f_L for protons $= \frac{e}{4\pi m_p} B$

$$\omega_L = 4.78 \times 10^7 \text{ rad s}^{-1} \text{ T}^{-1}$$

$$f_L = 7.8 \times 10^6 \text{ Hz T}^{-1}$$

$$= 7.8 \text{ MHz T}^{-1} \quad (\sim \frac{1}{1840} \text{ times})$$

- Basis of nuclear magnetic resonance (NMR) used in medical imaging
- e^- and protons accelerated ⇒ emit radiation
- 1 in GHz and 1 in MHz
- Can find radiation, find g_e and determine the kinds of atoms

Terms

1. Magnetisation (M)
   - dipole moment per unit volume

2. Magnetic Flux Density (B) Telsa = Wb/m^2
   - magnetic field developed inside material

3. Magnetic Field (H)
   - magnetic field applied to material

4. Magnetic Susceptibility (χ)

$$\chi_m = \frac{M}{H}$$

$$B = \mu_0(H+M) = \mu_0 H(1+\chi)$$

$$B = \mu_0(1+\chi)H$$

$$B = \mu_0\mu_r H \qquad \mu_r = 1+\chi$$

relative permeability

$$B = \mu H$$

Diagmagnetic Behaviour of Atomic Dipoles in Weak $\vec{B}_{ext}$

- First response of a material placed in weak $\vec{B}$ not to get magnetised
- Induced magnetic moment which opposes magnetic field through circulating currents
- Diamagnetic response of all materials, irrespective of type
- Due to precessing charge,

$$\mu_{ind} = -\frac{e}{2}\omega r^2$$

$$= -\frac{e}{2}\left(\frac{e}{2m}\right)Br^2$$

$$\mu = \frac{-e^2}{4m}r^2 B$$

If there are N_a atoms / volume and Z e^- per atom,

$$M = N_a Z \mu_{ind} = -N_a Z \frac{e^2}{4m} r^2 B = -N_a Z \frac{e^2}{4m} r^2 \mu_0 H$$

$$\chi_m = \frac{M}{H} = -N_a Z \frac{e^2}{4m} r^2 \mu_0$$

$$\chi_m = -\frac{N_a Z \mu_0 e^2 r^2}{4m}$$

negative, order of 10^{-5}

Types of Magnetic Materials

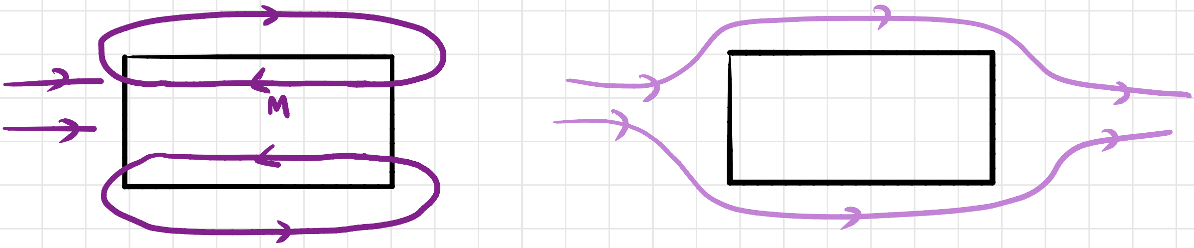
1. Diamagnetic
2. Paramagnetic
3. Ferromagnetic
4. Anti-ferromagnetic
5. Ferrimagnetic

DIAMAGNETIC MATERIALS

Lenz's Law at atomic level

- Materials with diamagnetic response even for high $\vec{B}$ (negative susceptibility $\sim -10^{-5}$)
- classical $\chi_{dia} = -\frac{Z N_a e^2 \mu_0}{6m} \langle r^2 \rangle$

  N_a = no. of atoms / volume
  $\langle r^2 \rangle$ = mean square value of radius
- χ_{dia} independent of temperature
- e^- shells / subshells are closed
- Superconductors - perfect diamagnetic
- $\vec{M}$ opposes $\vec{H}$ (magnetisation opposes applied field)



add up outside, cancel inside

Examples of Diamagnetic Materials

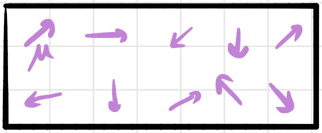
- water, wood, NaCl
- organic compounds (petroleum), plastics
- Cu, Hg, Au, Bi, Sb

highest χ in diamagnetic (-1.66×10^{-4})

PARAMAGNETIC MATERIALS

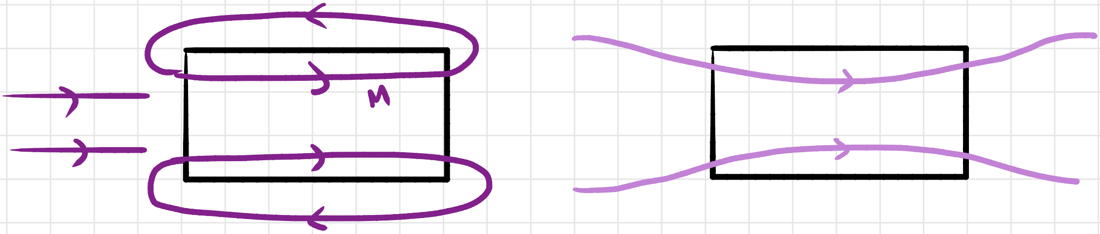
- Unquenched spin/angular momentum
- Atoms do not interact with each other

when $\vec{H}=0$



randomly oriented dipoles

- Thermal energy >> energy of interaction between dipoles ($\mu B \ll kT$) either in $\vec{H}$ or no $\vec{H}$
- In the presence of $\vec{H}$



cancels outside, adds up inside

- Due to unpaired electrons (net μ)

$$\mu = g_e \mu_B m_j$$

N atoms per unit volume,

$$M = N g_e \mu_B m_j \quad \text{(Probability factor)}$$

Temperature Dependence

- Two competing energies: temp. and interaction energy

1. $\Delta E = \mu B = -g_e \mu_B m_j \cdot B$

interaction energy

2. kT

Langevin - classical

- Maxwell-Boltzmann distribution (quantised - m_j)

$\leftarrow \mu B$

$$M = \frac{\sum -N g_e \mu_B m_j \, e^{\boxed{g_e \mu_B m_j \mu_0 H}/kT}}{\sum_{-j}^{j} e^{g_e \mu_B m_j \mu_0 H/kT}}$$

case I: $\mu B < kT$

case II: $\mu B > kT$

Case I: $\mu B < kT$

exponent < 1 $\quad$ $[x < 1)$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

$$M = \frac{N g_e \mu_B \sum_{-j}^{j} m_j \left(1 + \frac{m_j g_e \mu_B \mu_0 H}{kT}\right)}{\sum_{-j}^{j} 1 + \frac{m_j g_e \mu_B \mu_0 H}{kT}}$$

2j+1 orientations (-j ... 0 ... +j)

$$= \frac{N g_e \mu_B \sum_{-j}^{j} m_j + (m_j)^2 \frac{g_e \mu_B \mu_0 H}{kT}}{\sum_{-j}^{j} 1 + \frac{m_j g_e \mu_B \mu_0 H}{kT}}$$

$$\sum_{-j}^{j} m_j = 0 = j + (j-1) + \cdots + 0 + \cdots - (j-1) - j$$

$$\sum_{-j}^{j} (m_j)^2 = j^2 + (j-1)^2 + \cdots + (j-1)^2 + j^2$$

$$= 2 \sum_{1}^{j} (m_j)^2 = 2 \frac{(j)(j+1)(2j+1)}{6}$$

$$= \frac{j(j+1)(2j+1)}{3}$$

$$\sum_{-j}^{j} 1 = 2j+1$$

$$M = \frac{N g_e \mu_B \frac{(j)(j+1)\cancel{(2j+1)}}{3} \left(\frac{g_e \mu_B \mu_0 H}{kT}\right)}{\cancel{2j+1}}$$

$$= \frac{N (g_e)^2 (\mu_B)^2 \mu_0 H}{kT} \quad \frac{j(j+1)}{3}$$

$$= \frac{N (g_e)^2 (\mu_B)^2 \mu_0 H}{kT} \quad \frac{J^2}{3\hbar^2}$$

← total angular momentum not either spin or orbital

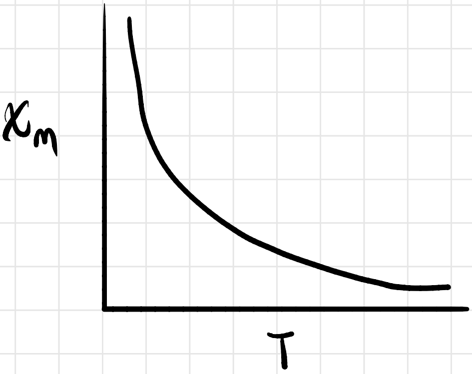
$$M = \frac{C}{T} H$$

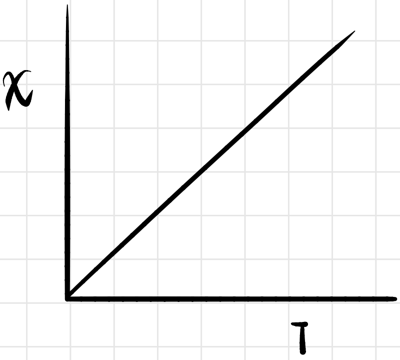
$$\chi_m = \frac{M}{H} = \frac{C}{T}$$

$$\chi_m \propto \frac{1}{T}$$

← succeptibility inversely proportional to T

curie's Law → for Paramagnetic materials





Case II : $\mu B > kT$

- Dipoles can interact with each other
- Magnetic interaction energy much greater than thermal energy

low temp OR high μB (2 diff. scenarios)

$$M = Ng_e\mu_B \frac{\sum_{-j}^{j} m_j\, e^{m_j g_e \mu_0 \mu_B H / kT}}{\sum_{-j}^{j} e^{m_j g_e \mu_0 \mu_B H / kT}}$$

$$x = g_e \mu_0 \mu_B H / kT$$

$$M = \frac{Ng_e\mu_B \sum_{-j}^{j} m_j\, e^{m_j x}}{\sum_{-j}^{j} e^{m_j x}}$$

(Similar to BBR, page 31 unit 1)

derivative of ln

$$M = Ng_e\mu_B \left(\frac{d}{dx} \left(\ln \left(\sum_{-j}^{j} e^{m_j x} \right) \right) \right)$$

$$= Ng_e\mu_B \left(\frac{d}{dx} \left(\ln \left[e^{jx} + e^{(j-1)x} + e^{(j-2)x} + \cdots + e^{-jx} \right] \right) \right)$$

$$= Ng_e\mu_B\left(\frac{d}{dx}\left(\ln\left(e^{jx}\left[1\underbrace{+e^{-x}+e^{-2x}+\cdots+e^{-2jx}}_{GP}\right]\right)\right)\right)$$

$$\frac{a(1-r^n)}{1-r}$$

$$= Ng_e\mu_B\,\frac{d}{dx}\left(\ln\left(e^{jx}\left(\frac{1(1-e^{-(2j+1)x})}{1-e^{-x}}\right)\right)\right)$$

$$= Ng_e\mu_B\,\frac{d}{dx}\left[\ln\left(\frac{e^{jx}-e^{-(j+1)x}}{1-e^{-x}}\right)\right]$$

$$= Ng_e\mu_B\,\frac{d}{dx}\left[\ln\left(\left(\frac{e^{jx}-e^{-(j+1)x}}{1-e^{-x}}\right)\frac{e^{x/2}}{e^{x/2}}\right)\right]$$

$$= Ng_e\mu_B\,\frac{d}{dx}\left[\ln\left(\frac{e^{(j+1/2)x}-e^{-(j+1/2)x}}{e^{x/2}-e^{-x/2}}\right)\right]$$

$$= Ng_e\mu_B\,\frac{d}{dx}\left[\ln\left(\frac{\sinh\left(\left(\frac{2j+1}{2}\right)x\right)}{\sinh(x/2)}\right)\right]$$

$$= Ng_e\mu_B\,\frac{d}{dx}\left[\ln\left(\sinh\frac{(2j+1)x}{2}\right)-\ln\left(\sinh\frac{x}{2}\right)\right]$$

$$= Ng_e\mu_B\left[\frac{\cosh\left(\frac{2j+1}{2}\right)x}{\sinh\left(\frac{2j+1}{2}\right)x}\cdot\left(\frac{2j+1}{2}\right)-\frac{\cosh(x/2)}{\sinh(x/2)}\left(\frac{1}{2}\right)\right]$$

$$= \frac{Ng_e\mu_B}{2}\left[(2j+1)\coth\left(\frac{2j+1}{2}\right)x-\coth\left(\frac{x}{2}\right)\right]$$

let $x = \frac{a}{j}$

$$M = \frac{N g_e \mu_B}{2}\left[(2j+1)\coth\left(\frac{2j+1}{2j}\right)a - \coth\left(\frac{a}{2j}\right)\right]$$

$$M = \overbrace{N g_e \mu_B j}^{\text{constant for a system } = M_s}\left[\frac{2j+1}{2j}\coth\left(\frac{2j+1}{2j}\right)a - \frac{1}{2j}\coth\left(\frac{a}{2j}\right)\right]$$

$$\underbrace{B_j(a)}_{\text{Brillouin function}} = \frac{2j+1}{2j}\coth\left(\frac{2j+1}{2j}\right)a - \frac{1}{2j}\coth\left(\frac{a}{2j}\right)$$

- Maximum magnetisation a material can have (when all are aligned) — saturation magnetisation
- $M = M_s \, B_j(a)$

$$B_j(a) = \overset{\text{max} = 1}{\frac{M}{M_s}}$$

$B_j(a)$ can have a max. value of 1

Brillouin Function $B_j(a)$

Case I : $j \rightarrow \infty$

- if j is very large, $2j >> 1$

$$B_j(a) = \coth(a) - \left(\frac{1}{a}\right)\frac{a}{2j}\frac{\cosh(a/2j)}{\sinh(a/2j)}$$

$$- \frac{1}{a} \quad \frac{\cosh\cancelto{1}{(a/2j)}}{\cancelto{1}{\frac{\sinh(a/2j)}{(a/2j)}}}$$

$$B_j(a) = \coth(a) - \frac{1}{a} \longleftarrow$$

Langevin's function (classical)

- Langevin's method: dipoles orient in all possible orientations ($-\pi$ to $+\pi$)
- Implication of j: for j, m_j : $-j$ to j $(2j+1)$ orientations
- As $j >>$ large, $2j+1 \longrightarrow$ large $\longrightarrow$ Langevin's
- Langevin's function

$$L(a) = \frac{a}{3} - \frac{a^3}{45} + \frac{2a^5}{945} - \ldots$$

$$a = xj$$

$$x = \frac{g_e \mu_0 \mu_B H}{kT}$$

$$\therefore a = \frac{g_e \mu_0 \mu_B H j}{kT}$$

small a / small x means high temperature

$\therefore$ higher order terms can be neglected

$$\therefore L(a) = \frac{a}{3}$$

- For large values of j and small values of a,

$$B_j(a) = \frac{a}{3} = \frac{M}{M_s}$$

$$M = M_s \frac{a}{3} \quad ; \quad M_s = N g_e \mu_B j$$

$$M = N g_e \mu_B j \, \frac{g_e \mu_0 \mu_B H j}{3kT}$$

$$M = \frac{N g_e^2 \mu_B^2 \mu_0 H j^2}{3kT}$$

Similar to Curie's Law

Curie's Law

$$M = \frac{N g_e^2 \mu_B^2 \mu_0 H}{kT} \quad \frac{j(j+1)}{3}$$

At high values of j, $j(j+1) \approx j^2$

- For any material, at large values of j and high temperatures, paramagnetic behaviour still persists

$$\chi = \frac{N g_e^2 \mu_B^2 \mu_0}{kT} \quad \frac{j^2}{3}$$ (positive value)

Case II : $j = 1/2$

- j is purely spin ; no orbital

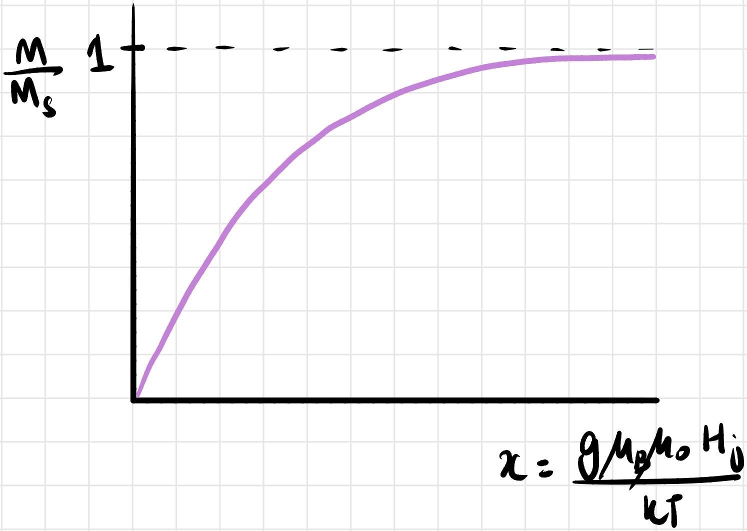
$$B_j(a) = \frac{2j+1}{2j} \coth\left(\frac{2j+1}{2j}\right)a - \frac{1}{2j} \coth\left(\frac{a}{2j}\right)$$

$$B_{1/2}(a) = 2 \coth(2a) - \coth a$$

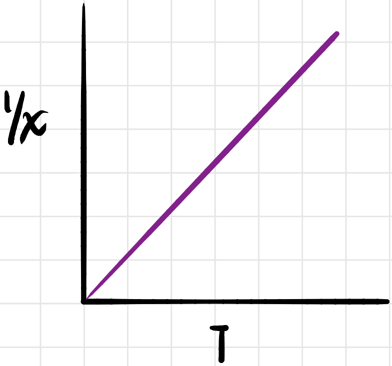
$$= \frac{2(1+\tanh^2 a)}{2 \tanh a} - \frac{2}{2 \tanh a}$$

$$B_{1/2}(a) = \tanh a$$

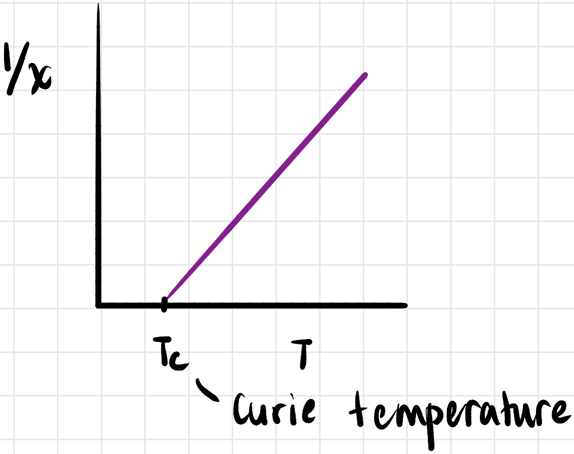
$M = M_s \tanh a$



$1/\chi$ vs T



pure paramagnetic



Not pure paramagnetic

Weiss Molecular Field Theory

- Dipole creates magnetic field surrounding it
- Influences neighbouring dipoles

$$\chi_{actual} = \frac{M}{H_{tot}} = \frac{\text{magnetisation}}{\text{external + internal field}}$$

$$\chi_{actual} = \frac{M}{H + \lambda M} = \frac{C}{T}$$

net contribution of neighbours

$$C(H + \lambda M) = MT$$

$$M(T - \lambda C) = CH$$

$$\chi_m = \frac{M}{H} = \frac{C}{T - \lambda C}$$

measured experimentally

λC must have units of temp

$$\chi_m = \frac{C}{T - T_c}$$

- Materials have paramagnetic behaviour above a temperature T_c (Curie temperature)

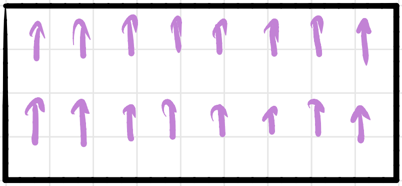
- For purely paramagnetic substances, $T_c = 0$ and graph passes through origin

- Ferromagnetic substances at high temperatures: thermal energy ≈ field energy and leads to random orientations of dipole

FERROMAGNETIC MATERIALS

- $\chi_m = +ve$, 1 to $\sim 10^5$
- For $T < T_c$, ordered dipoles

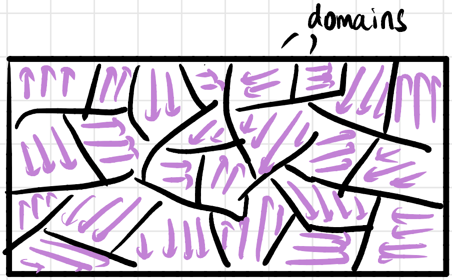
strong interaction



pure single crystal

(too expensive)

- Even for $H=0$, M is nonzero $(M>0)$
- Adjacent dipoles orient in the same direction



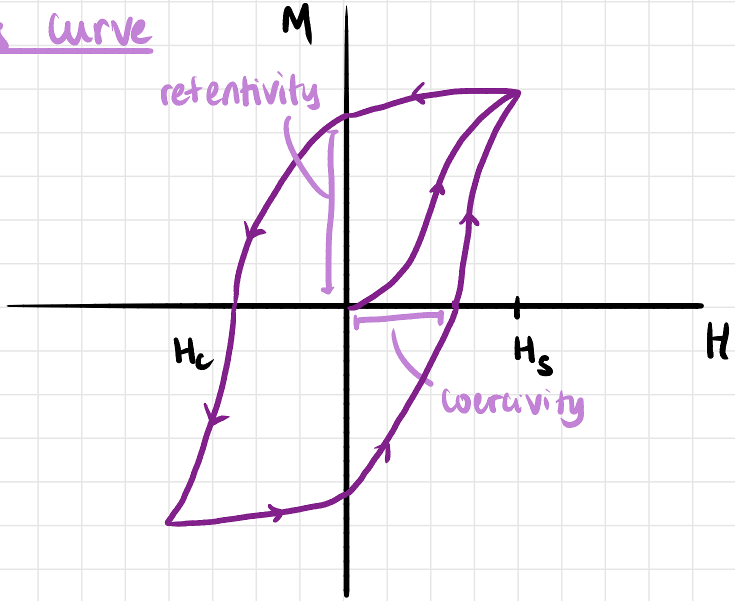
density close to crystalline density

sintered polycrystalline material

(high temp, pressure)

- For $H=0$, $M=0$ (net $M=0$)
- Need to be magnetised at high fields

Hysteresis Curve

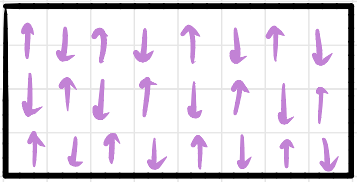


- Area under curve — energy lost
- Hysteresis — memory
- Hard discs - ferromagnetic materials (thin films) magnetic memory
- Floppy drive ⟶ 1 TB hard disc
- Domain sizes — 50-100 nm ⇒ can individually manipulate dipoles
- Formatting discs: $-H_c$ or $+H_c$ used
- High T_c materials preferred
- Partly filled 3d-4d shells filled using Hund's Rule
- Fe, Ni, Co, Gd, Dysposium, Samarium, Neodymium w Co

ANTIFERROMAGNETIC MATERIALS

- for $T < T_N$ Neel temperature
- also ordered dipoles ; magnetisation is 0

antiparallel dipoles (equal)



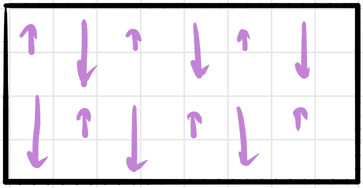
highly strong coupling between neighbours

- magnetic insulators (can be used on ferromagnetic) due to high inertia
- also used in magnetic memory (in read-write head)
- MnO, NiO, CoO

FERRIMAGNETIC MATERIALS

- for $T < T_c$
- Ordered dipoles, still destructive
- Domains can be flipped at high frequency
- Antiparallel but unequal magnitudes

net dipole alignment



- Sub-lattices have different magnetic moments

- Cations of 2 or more types, diff magnetic moments

- $NiFe_2O_4$, $CoFe_3O_4$, $BaFe_{12}O_{19}$

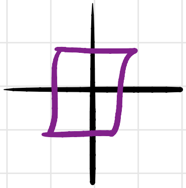
* refer pdf for nature of graph

Hard & Soft Magnetic Materials

- Ferromagnetic Materials, $T < T_c$

- Based on nature of hysteresis curve

Hard Magnetic Materials



- More squarish → $H_s \approx H_c$

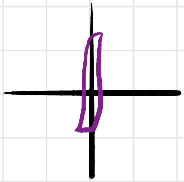
- Hard discs use — better permanent magnets

- Difficult to magnetise and demagnetise (large area under curve)

- Ideal for memory devices

Soft Magnetic Materials

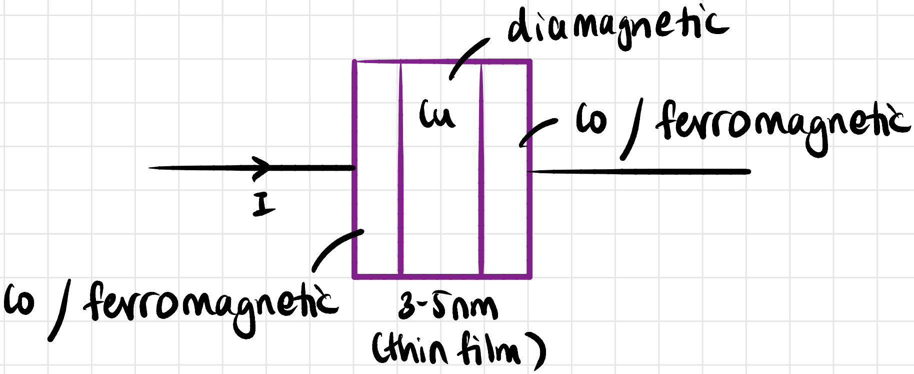
- Thin hysteresis curve



- Easier to magnetise and demagnetise (small coercivity)
- Good candidate for electromagnets, transformers, motors, generators
- High frequency switching (AC : 50 Hz)
- Low losses (low area under curve) and low heat losses
- Ferrites – used in high frequency switching

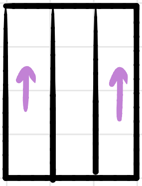
Magneto Resistance

- Change in resistance in the presence of an external magnetic field (small change)
- Resistance depends on spin orientation
- Device – Giant Magneto Resistance Device

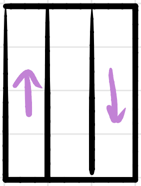


$$R = \frac{V}{I}$$

- Resistance very sensitive to magnetisation state
- Parallel resistance: both dipole moments in same orientation (low resistance)



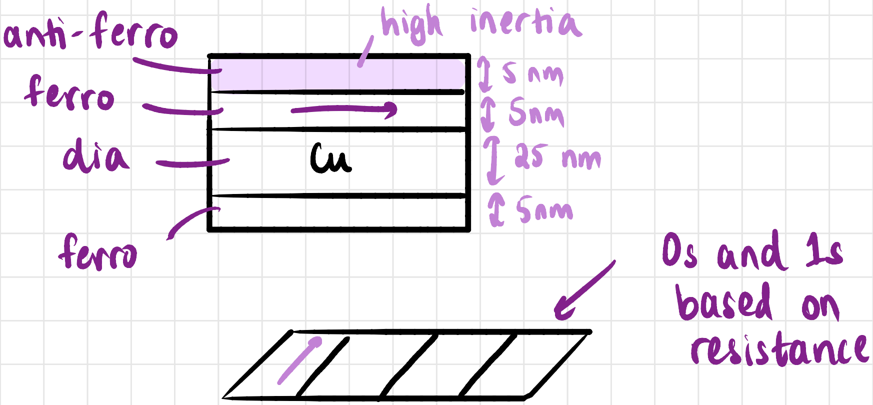
- Antiparallel resistance: dipole moments in opposite directions (high resistance)



- By switching the orientation of one of the layers, the resistance can be controlled
- GMR coefficient

$$GMR = \frac{R_{\uparrow\downarrow} - R_{\uparrow\uparrow}}{R_{\uparrow\downarrow}} \longrightarrow \text{up to } 50\%$$

- Modified construction: 4 layers



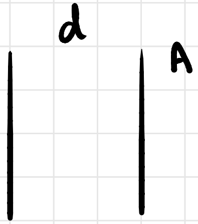
- A pin layer (antiferromagnetic) layer is placed over one ferromagnetic layer to maintain its orientation (insulator)
- A thick diamagnetic layer is placed between the two ferromagnetic layers
- The other ferromagnetic layer is the free layer

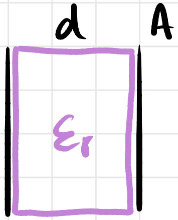
Ⅱ

DIELECTRICS

- Charges cannot be continuously displaced in such media
- Insulators are similar
- Dielectrics: charges may be displaced by small distances
- Used in capacitors

$$C_0 = \frac{\varepsilon_0 A}{d}$$



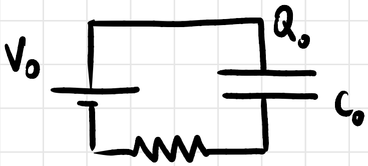


$$C = \varepsilon_r \frac{\varepsilon_0 A}{d}$$

$$C = \varepsilon_r C_0$$

$$\varepsilon_r = \frac{C}{C_0}$$

Charging



$$Q_0 = C_0 V_0$$

$$E_0 = \frac{V_0}{d}$$

Dielectric without power supply

$$C = \varepsilon_r C_0$$

$$Q_0 = CV$$

$$\therefore V = \frac{V_0}{\varepsilon_r}$$

$$E = \frac{V}{d} = \frac{V_0}{\varepsilon_r d}$$

$$E = \frac{E_0}{\varepsilon_r}$$

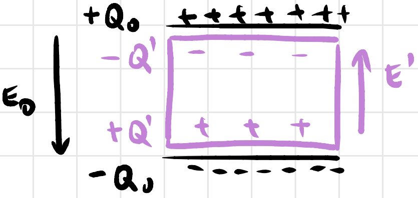
Dielectric with power supply

$C = \varepsilon_r C_0$

$V = V_0$

$Q = \varepsilon_r Q_0$

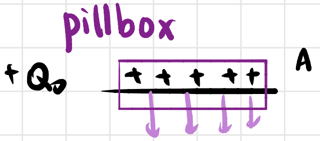
$E = E_0$



$E = E_0 - E'$

Using Gauss' Law

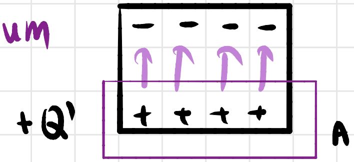
metal plate



$$E_0 = \frac{Q_0}{A\varepsilon_0} = \frac{\sigma_0}{\varepsilon_0}$$

dielectric medium

(assume uniform $\vec{E}'$)



surface charge density

$$E' = \frac{Q'}{A\varepsilon_0} = \frac{\sigma'}{\varepsilon_0}$$

$$\sigma' = \frac{Q}{A} \cdot \frac{d}{d} = \frac{\text{dipole moment}}{\text{volume}}$$

= polarisation in the material

$\sigma' = P$

- Note: in magnetism, flux could increase or decrease with media

- In dielectrics, polarised field always opposes

$$E = E_0 - E'$$ (opposite directions)

$$E = E_0 - \frac{P}{\varepsilon_0}$$ polarisation

We define $\varepsilon_r = \frac{E_0}{E} = \frac{C}{C_0} = \frac{V_0}{V}$

$$\therefore E = \varepsilon_r E - \frac{P}{\varepsilon_0}$$

$$P = \varepsilon_0 (\varepsilon_r - 1) E$$ polarisation of material in the presence of an electric field

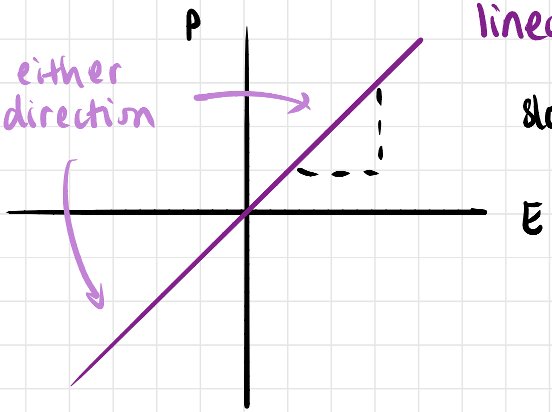
$$P = \chi_d \varepsilon_0 E$$ $\chi_d = \varepsilon_r - 1$

$$P \propto E$$ dielectric susceptibility

Similar to Magnetisation,

$$M = \chi_m / \mu_0 B$$ $\chi_m = \mu_r - 1$

magnetic susceptibility

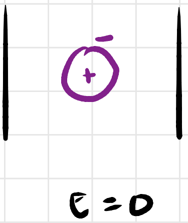


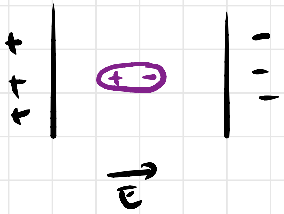
linear dielectric

slope = $\chi_d \varepsilon_0$

Atomic Level

- In the absence of an external $\vec{E}$, atoms are neutral
- centre of +ve charges = centre of -ve charges = centre of atom (nucleus) $\Rightarrow$ no dipoles exist
- In the presence of an external field, charges get displayed slightly (spherical symmetry lost)
- Small dipoles formed





- Polarisability (of atoms) = dipole moment per atom (α_e)

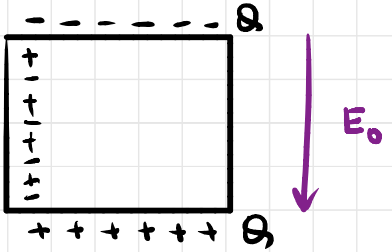
$$P = N\alpha_e E$$ (N = no. of atoms / vol)

$$N\alpha_e = \varepsilon_0(\varepsilon_r - 1)$$

atomic polarisability

$$\alpha_e = \frac{\varepsilon_0(\varepsilon_r - 1)}{N}$$

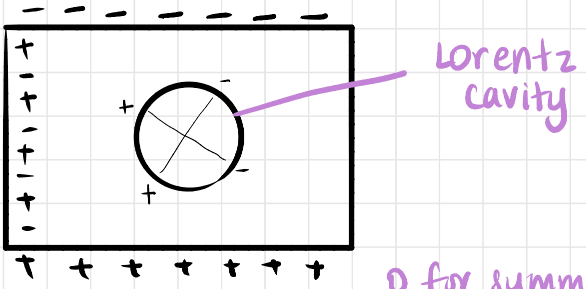
Dielectrics in External Field



fcc crystal: remains symmetric when viewed from any direction

- Internal dipoles also contribute to net $\vec{E}$
- Local field: field at a sight, slightly diff from $\vec{E}_{avg}$ due to local effects

(fcc)



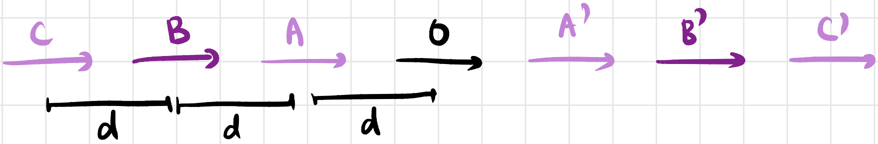
0 for symmetric (fcc)

$$E_{loc} = E_0 + E_{\substack{surface \\ (depolarising)}} + E_{\substack{Lorentz \\ cavity}} + E_{other\ dipoles}$$

inside cavity

$$E_{loc} = E_0 + E_1 + \cancelto{0}{E_2} + E_3$$

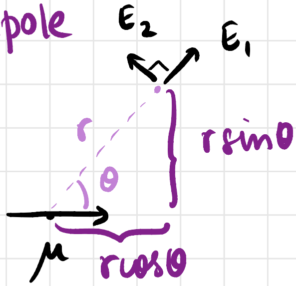
Consider a line of dielectrics (fixed)



Find contributions of A and A' on O

Pairs of dipoles at uniform distances

$\vec{E}$ due to dipole



$$E_1 = \frac{2\mu\cos\theta}{4\pi\varepsilon_0 r^3}$$

$$E_2 = \frac{\mu\sin\theta}{4\pi\varepsilon_0 r^3}$$

Field at O due to A

$$E_A = \frac{2\mu}{4\pi\varepsilon_0 d^3}$$

Field due to A'

$$E_{A'} = \frac{2\mu}{4\pi\varepsilon_0 d^3}$$

Net field at O due to A and A'

$$E_{AA'} = \frac{\mu}{\pi\varepsilon_0 d^3}$$

The total electric field at o due to pairs of dipoles equidistant from O

$$E_3 = \frac{\mu}{\pi\varepsilon_0 d^3} + \frac{\mu}{\pi\varepsilon_0 (2d)^3} + \dots$$

$$E_3 = \frac{\mu}{\pi\varepsilon_0 d^3} \boxed{\sum_{n=1}^{\infty} \frac{1}{n^3}}$$

Apéry's constant

≈ 1.2

constant (geometry)

$$E_3 = \frac{1.2\,\mu}{\pi\varepsilon_0 d^3}$$

$$\frac{\text{dipole moment}}{\text{volume}} = P \text{ (polarisation)}$$

$$= \frac{\gamma P}{\varepsilon_0} = \frac{P}{3\varepsilon_0}$$ (for cubic material)

The contribution of $E_{AA'}$ to E_3 is the most significant one (~83%)

The contribution of $E_{BB'}$ to E_3 is 1/8th $E_{AA'}$'s (~10%)

The contribution of $E_{CC'}$ to E_3 is ~ 3%

depolarisation

$$\therefore E_{loc} = \underbrace{E_0 + E_1}_{E} + \frac{P}{3\varepsilon_0}$$

polarisability　　　　average (bulk)

$$P = N\alpha_e E_{loc} = \varepsilon_0(\varepsilon_r - 1)E$$

$$N\alpha_e\left(E + \frac{P}{3\varepsilon_0}\right) = \varepsilon_0(\varepsilon_r - 1)E$$

$$P = \chi_d \varepsilon_0 E = \varepsilon_0(\varepsilon_r - 1)E$$

$$N\alpha_e\left(\cancel{E} + \frac{(\varepsilon_r - 1)\cancel{E}}{3}\right) = \varepsilon_0(\varepsilon_r - 1)\cancel{E}$$

$$\frac{N\alpha_e}{\varepsilon_0}\left(\frac{3 + \varepsilon_r - 1}{3}\right) = \varepsilon_r - 1$$

Clausius-Mossotti Relation

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha_e}{3\varepsilon_0}$$

Relates microscopic and macroscopic parameters (solids)

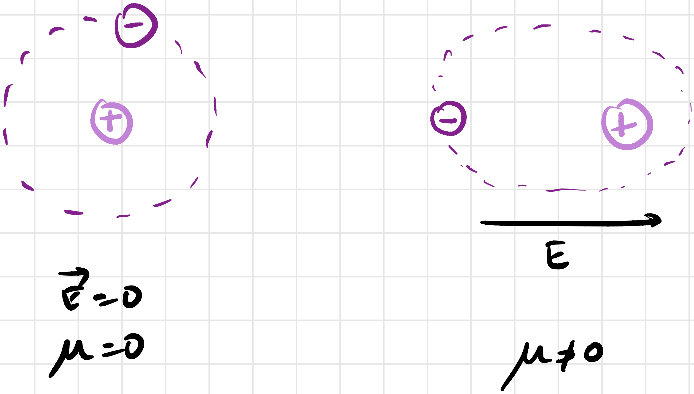
For gases, d is not fixed and we use the conventional relationship (page 34)

$$\frac{N\alpha_e}{\varepsilon_0} = \varepsilon_r - 1$$

Polarisation Mechanisms

1. Electronic Polarisation

- Atom subjected to electric field : separation of +ve and -ve charge centres
- Dipoles induced



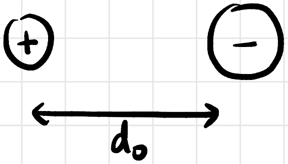
- Limit of field that can be applied : ionisation

$$\alpha_e = 4\pi\varepsilon_0 R^3$$

- Mechanism independent of temperature
- Atomic level mechanism

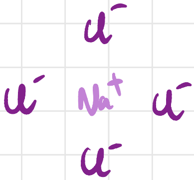
2. Ionic Polarisation

- Materials bonded ionically (permanent dipoles)



$\mu \neq 0$

- Crystal structure: fcc



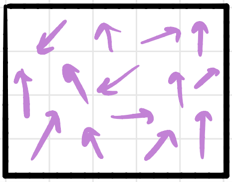
$E = 0$

net $\mu = 0$

- When $\vec{E}$ applied, dipole induced ($\mu \neq 0$)
- Polarisability due to displacement of ions from equilibrium (Hooke's Law)
- Polarisability α_i is dependent on Young's Modulus of elasticity Y
- Y is temperature-independent ⇒ mechanism is temperature independent

3. Orientation Polarisation

- Applicable to polar materials



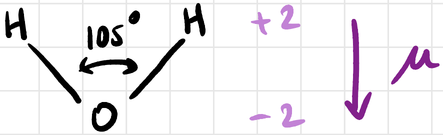
$\mu \neq 0$

$E = 0$

- If $kT >>$ large, $\mu = 0$ (similar to paramagnetic materials)

$$P = \frac{N\mu^2 E}{3kT}$$

Water molecules



$\mu \longrightarrow \varepsilon_r = 80 \longleftarrow$ cannot be made useful; we get conductivity and cannot be capacitor

- Dipoles exist when $\vec{E} = 0$
- External field can orient dipoles at lower temperatures
- Temperature dependent.

4. Space Charge Limited Polarisation

- To make capacitor, multiple granules of system mixed with low-melting oxides (Sb_2O_2, Bi_2O_2)
  glasses
- Low-melting oxides have diff conducting characteristics (phases)
- They do not stay together at low temp.
- Barium Titinate gets thin layer of glasses
- Different conducting phases ⇒ phase charge polarisation (artificial polarisation)

Non-Linear Dielectrics

- $\frac{\partial P}{\partial E} \neq$ constant

mech $\frac{\partial P}{\partial S}$

thermal $\frac{\partial P}{\partial T}$

32 electrostrictive classes → 21 non centre of symmetry → 20 Piezo electric classes → 10 pyro electric classes

32 electrostrictive classes ↓ 11 centre of symmetry classes

21 non centre of symmetry ↓ 1 non-piezo electric class

20 Piezo electric classes ↓ 10 non-Pyro electric classes

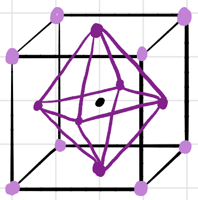
10 pyro electric classes ↓ 10 Ferro electric classes

$\frac{\partial P}{\partial E}$

electric

Centro Symmetric System

- $BaTiO_3$ — standard material
- T > 120°C , system in cubic form



● – Oxygen $(6 \times 1/2 = 3)$
● – Barium $(8 \times 1/8 = 1)$
● – Titanium $(1 \times 1 = 1)$

- Oxygen : $3 \times O^{2-} = 6^-$
  Barium : $1 \times Ba^{2+} = 2^+$
  Titanium : $1 \times Ti^{4+} = 4^+$
- In this case, material is centro symmetric as +ve and -ve charges coincides
- Material is paraelectric (linear response)

Non-Centro Symmetric System

Case I

- T between 5° and 120°
- Material gets into tetragonal phase (elongated cube)
- Ti^{4+} no longer at centre of crystal
- Charge separation (fraction of Å)

- $\mu \neq 0$ when $E = 0$
- Can manipulate position of Ti^{4+} using mechanical stress, thermal energy, electric field → polarisation

Case II

- $T < 5°C$ and $T > -90°C$
- Orthogonal system $(a \neq b \neq c)$

Case III

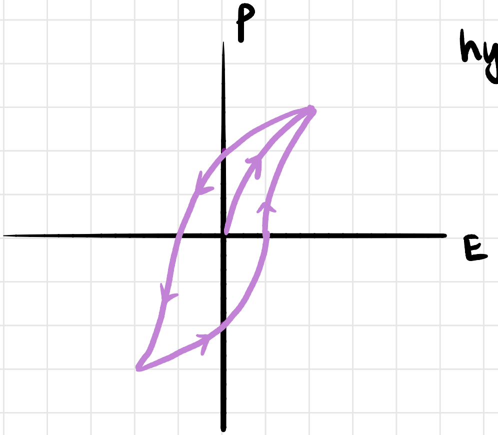
- $T < -90°$
- Orthorhomic system

FERROELECTRIC MATERIALS

- Characteristics very similar to ferromagnetic materials
- Nothing to do with iron

polycrystalline material

$E = 0$
$\mu = 0$



hysteresis

- Good for memory
- Dynamic RAM in cellphones
- 1 — polarisation $(+E_s)$ $\uparrow_{pr}$
  0 — polarisation $(-E_s)$ $\downarrow_{pr}$
- To erase, E_c (+ or −) used

PIEZOELECTRICS

- Non centro-symmetric system

direction of stress

- Response to mechanical stress $\left(\frac{\partial P}{\partial s}\right)$ — highly directional
- Volume changes → polarisation (per unit volume) changes
- Mechanical stress creates electric potential difference
- Indirect piezoelectric effect : electrical → mechanical (Crystal oscillates due to $\vec{E}$)

- If frequency of $\vec{E}$ = natural frequency of crystal, mechanical standing waves formed (resonance)

Tensor

- χ is tensor
- not a vector, nor a scalar
- collection of points showing some directionality

- PVDF - plastic film: vibrates in wind and generates microvolt power

- When crystal strained, electric field produced and potential difference developed

- Range of applications
  - → sensitive chemical and biological sensors
  - → pressure measurement to study explosives, IC engines, etc.
  - → electric igniters
  - → electronic drum pads (transducers)

PYROELECTRICS

$T > T_c$,
↓ centro symmetric

- Non centro symmetric crystals

- Responce to thermal stress $\left(\frac{\partial P}{\partial T}\right)$ for $T <$ Curie temp

- Pyroelectric coefficient p_i

$$p_i = \frac{\partial P}{\partial T}$$

- Heat sensing element – fire alarm
- Device to monitor capacitance